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Mathematics: The ongoing crisis

Rhonda Farkota

In December 2016 the Australian Council for Educational Research (ACER) released the Programme for International Student Assessment (PISA) report on the science, reading and mathematics skills of Australian 15-year-olds, illustrating that Australia is not only slipping backwards relative to other countries, but we are getting worse at preparing our students for the everyday challenges of adult life. The Director of Educational Monitoring and Research Dr Sue Thomson stated that evidence from the 2015 Programme now makes clear that the science, reading and mathematics achievement of Australian students is in absolute decline, and ACER Chief Executive Officer, Professor Geoff Masters stated that the decline was dramatic.

It's dispiriting to note that the math crisis that has existed in Australian schools for such a long time has continued to deteriorate. As this writer noted more than a decade ago: There are many possible reasons why Australian students are failing in mathematics, but most of them are related to curriculum and methods of teaching rather than any student lack of capacity to learn. Equally dispiriting: There is still no consensus on how the problem should be addressed.

With respect to mastering mathematics, it's essential to bear in mind one important, long-recognised fact, and that is a student of average ability requires many presentations of a new concept before learning and remembering it. Ergo, genuine mastery of both basic skills and problem-solving can only come about with constant practice.

The old adage about crawling before walking is appropriate here, for while it's all very well to encourage students to solve problems by reasoning and reflecting, it's important for them to first acquire the basic skills that provide the scaffold upon which problem-solving skills are built. Problem-solving skills almost invariably operate from a knowledge base that has been acquired through practice, but it is actually when the knowledge in a discipline is being acquired that the foundations for effective problem-solving are being laid. Fluency and automaticity, which are by-products of mastery, come about when this knowledge base can be instantly tapped into without any great mental effort, giving students the opportunity to maximise their mental powers on more complex tasks.

Since the days of Plato, the big education debate has revolved around which is better: Student-directed learning or Teacher-directed learning?

The author, in her doctoral research conducted a comprehensive review of the relevant research and literature, and reached the inescapable conclusion that some skills were better acquired through one approach, and some through the other. When it came to the employment and cultivation of higher order skills, where reasoning and reflection were required, it was clear that a student-directed approach to learning was more appropriate. But when it came to the acquisition of basic skills, the empirical evidence unequivocally showed that a teacher-directed approach was better suited.

Bearing the above in mind, the Math Mastery Series (MMS) was specifically designed for the Australian classroom. Employing its own distinctive Direct Instruction (DI) model, it strikes a balance between teacher-directed learning and student-directed learning.

Daily lessons are briskly delivered in small incremental portions, with direct explanation and daily systematic review of student work. Ideally, unobtrusive assessment should be an integral part of the learning process. It is a fact of life that few of us enjoy sitting down to examinations, so these lessons were designed to test the students, without being seen by them as formal testing. Student personal analysis of incorrect responses given during a lesson, however, provides teachers with reliable diagnostic information better than any that can be acquired from a formal test situation. This is crucial to student success because it allows for teacher feedback to specifically target individual student misunderstanding.

It is well accepted that to perform a task competently students require not only the requisite skills, but also the self-belief in their ability to implement performance. In the learning process, this is termed *self-efficacy*, and when laying the foundational skills in mathematics, or for that matter any academic discipline, it is important that student self-efficacy be accommodated. Students with low self-efficacy in a particular skill area are reluctant to engage in tasks where those skills are required, and if they do, they are more likely to quit when encountering difficulty. Daily feedback on their performance makes students aware of their progress, which strengthens their self-efficacy, and at the same time enhances their academic achievement. As students engage in the daily lessons they learn which actions produce desirable results, which in turn motivates them to persevere towards mastery.

Student self-evaluation of progress is an integral and ongoing component of the MMS. Because the tasks gradually increase in difficulty, students have clear criteria by which they can independently assess their performance and gauge their progress. As they progress they acquire more skills and become more proficient at the self-evaluation process.

A perennial problem for teachers

at the start of each school year is the diverse, and all too often, inadequate, academic standard of their new class. The traditional practice of teaching mathematics in single topics creates many problems for students. Presenting them with a heap of new information in one hit, expecting them to master it; then move onto another, often unrelated topic, master that too, and so on, is a big ask. The problem is compounded when students are not re-familiarised with the topics throughout the year.

The MMS circumvents this problem by running concurrent strands. Because the strands are run concurrently, students are soon familiar with the many connections existing between the various math disciplines, and become fluent in applying them. Once foundations to the core areas have been laid and tested, they are built on with small precise portions. None of this incremental information is left on the shelf. Students move on to questions that gradually increase in complexity, all the while relying on the skills they have acquired along the way. These questions shift from abstract numbers to real-life situations so students see the relative worth of mathematics in situations that arise in the everyday world. Students quickly learn that everything they are taught is important; everything they learn is revisited, developed further, and gradually integrated into the broad mathematical landscape. This gradual and consistent development of skills is one of the key elements of teaching to mastery.

At the outset, this article alluded to the ongoing math crisis, the ancient Student-directed learning versus Teacher-directed learning debate, and the author's doctoral research into those matters; and although the MMS and the thesis were written twenty years back, the concluding words to that thesis are every bit as relevant today:

> *In conclusion it seems to this writer there is a dire urgency for the academics of the education world to put less emphasis on the ideology they feel most comfortable with and have a long hard look at the evidence. In the light of the research reviewed in this thesis it is impossible to deny the need for structured teaching in certain important circumstances just as it is impossible to deny the potential benefits to be had from student-directed learning in appropriate circumstances. If we are to provide the children of this nation with the best possible education, clearly, a balance must be achieved between teacher-directed learning and student-directed constructivist approaches — and for the children's sake it must be achieved soon. Further, it is submitted that the fitness for purpose principle …should be the guiding light when it comes to setting that balance. Unfortunately there is no one stop shop — no panacea when it comes to education — it just isn't that simple.*

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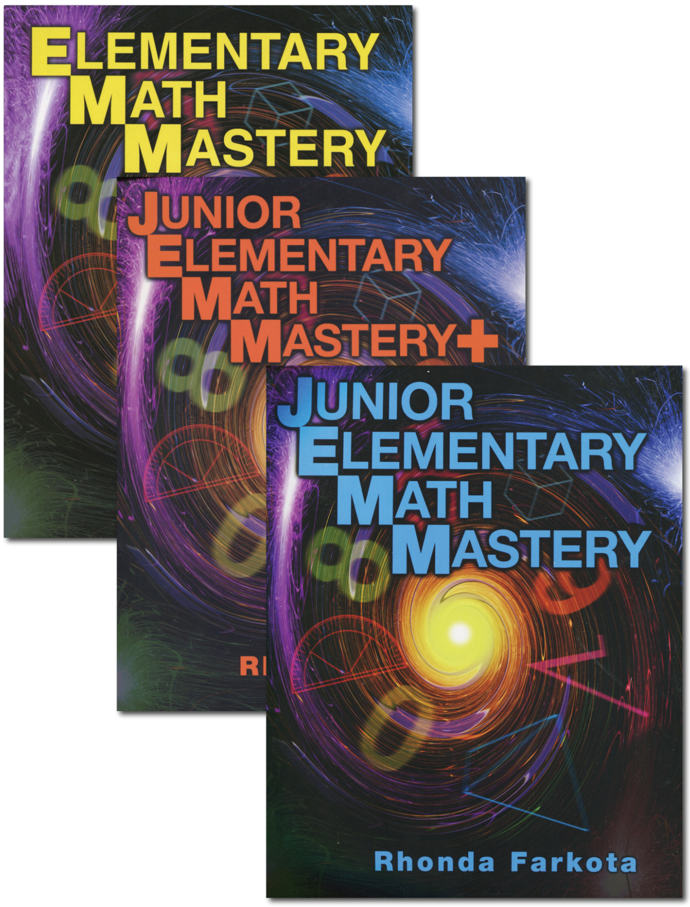

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Dyscalculia and Difficulties with Mathematics

Tanya Forbes

Dyscalculia is an inherited neurological condition that affects the acquisition of skills in mathematics. Difficulties in numeracy are thought to be as widespread as literacy difficulties; however, there has been much less research on dyscalculia than dyslexia (Butterworth, 2004). The incidence of dyscalculia is currently estimated to be between 6-7% of the general population (Callaway, 2013), although the figure varies because researchers use different criteria to define severe mathematical difficulties.

Generally, students with dyscalculia will lack number sense: they will be unable to grasp number concepts, will have problems learning number facts, will have trouble performing simple calculations, and will be unable to apply their mathematical knowledge to solve problems. These students tend to be resistant to good instruction and may be unable to retain and apply what they have learned.

Every student's profile will be different, but students will typically have difficulties with:

- learning to count. Students may use immature strategies to calculate such as counting by ones, often with their fingers.
- recognising number symbols
- understanding mathematical operations and performing calculations
- learning and recalling basic mathematical facts, particularly the times tables
- recognising patterns in numbers
- understanding the structure of numbers such as place value and grouping
- telling the time
- reading and interpreting graphs and charts.
- grasping abstract concepts like multi-step algorithms, fractions and algebra
- applying mathematical concepts to everyday life, such as budgeting and time management skills (Understood, 2017)

As well as basic difficulties with number sense, students may experience other limitations that impact on their success in the mathematics classroom:

Language processing problems: Accurate word reading and good comprehension are essential to understand word problems and recognise relevant information in a question. Many words have several meanings so students need a comprehensive vocabulary to understand the precise meaning of mathematical terms (Garnett, 1998).

Visuo-spatial problems: Some students may have difficulty making sense of visually presented information and visualising concepts. Visual-spatial difficulties can result in a poor sense of direction, mixing up left and right, confusion between 'less' and 'more', and trouble with measurement (Szucs, Devine, Soltesz, Nobels & Gabriel, 2013).

Memory difficulties: All aspects of memory are involved in mathematics. Short term memory allows recall of sequences and procedures. Working memory provides temporary storage of facts and figures while performing a calculation. Long term memory allows immediate recall of facts and information. If one area of memory is weak, then that will have a significant impact on maths performance. Often a slow processing speed will accompany problems with memory (Lee Swanson, et al. 2001).

Attention deficits: A high level of sustained concentration is required to learn maths concepts. Students need to maintain attention to effectively process new information, to recall maths facts and to self-monitor for careless mistakes. Attention deficits can have a significant impact on mathematical learning. Mental calculations follow an 'order of operations' and problem solving is often a multistep process. Students need to sequence each step in the right order to calculate the correct answer

Anxiety: Poor performance in maths can have a significant emotional impact. Constant failure leads to a loss of confidence, low self esteem and anxiety. When a student has a negative experience, they feel discouraged and this can lead to avoidance. High anxiety can have a direct impact on working memory leading to a further drop in performance. Some students soon believe that maths is difficult, so they give up and disengage from learning (Ashcraft, 2002).

Ways to support students with numeracy difficulties

Avoid rote learning, rapid fact recall and repetitive drills – these are empty of meaning and difficult to remember. Instead, we should teach the relationships between maths facts to develop students' understanding of basic number and operation concepts. By teaching for meaning, students are able find a solution using logic and reasoning when their memory fails them.

Pre-teach and review relevant skills and introduce new vocabulary to ensure that students have a correct understanding of the essential sub-skills required to complete the task. Build on prior knowledge by connecting new information to background knowledge to provide the student with a solid foundation to build knowledge and skills. Develop a conceptual framework to store new learning and provide strategies so information is easily retrieved and applied.

Use simple, clear and concise language during explanations – always focus on critical content and break complex skills into small manageable steps. Provide a step-by-step demonstration of problem solving - model the skill by thinking aloud as you solve the problem. Students will learn to 'self talk' through the process. Then lead students through a range of worked examples, using prompting and questioning to actively engage students in the problem solving process.

Use manipulatives (concrete materials such as blocks) and visual representations (drawings or figures) to help students to learn basic maths operations, solve story problems and master abstract concepts like fractions and algebra, as well as to explain, simplify and clarify problems. Manipulatives and diagrams function as cognitive tools to connect students to concepts: they may make difficult ideas understandable, complex problems solvable and abstract concepts tangible (Butler, Miller, Crehan, Babbit & Pierce, 2003; Cass, Cates, Smith & Jackson, 2003; Sowell, 1989; Witzel, Mercer & Miller, 2003).

Encourage students to verbalise their problem solving process – provide opportunities for students to explain concepts, describe procedures and discuss the ideas they are developing. Questioning helps students make sense of information and provides teachers with insight into learning. Frame questions to help students' gain new understanding, consolidate their learning and monitor their progress. Use feedback to check for understanding, and to provide positive reinforcement and corrective feedback. Reinforce that mistakes are an important part of the learning process – we learn from our mistakes.

Give students plenty of time. Students will need extra time to learn skills, process information and perform calculations. Allow plenty of opportunities to practise to consolidate learning and achieve mastery. Practice will increase fluency in processing, improve retention of information, facilitate recall and develop understanding. Students with learning difficulties will need more practice.

Provide scaffolding to promote success and build confidence. When students demonstrate mastery, you can gradually increase task difficulty as you decrease the level of guidance. Most importantly, teach for success so students see themselves as competent problem solvers. Then they will be more willing to attempt tasks and persevere with difficult problems.

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The cost of not counting: Developmental Dyscalculia and low numeracy

Ann Williams

Awareness amongst educators of the specific learning disability *dyscalculia* is limited compared to awareness of the better understood condition *dyslexia*. Perhaps as a consequence, dyscalculia attracts far less research funding. According to Butterworth, Varma and Laurillard (2011), the National Institutes of Health (NIH) had spent $107.2 million funding dyslexia research in the United States since 2000, but had spent only $2.3 million on dyscalculia research. This is despite the prevalence of the two conditions being similar. This apparent lack of awareness and action may have consequences for both the individual and the community.

International comparative measures such as the *Programme for International Student Assessment* (PISA) and *Trends in Mathematics and Science Study* (TIMMS) show a decline in mathematics performance in Australia over a period of years. This trend is particularly notable amongst disadvantaged students, with up to 52% of disadvantaged students in Year 4 having difficulties with mathematics (Thomson, 2016; Thomson et al., 2012). While some students may not have been exposed to the necessary high quality teaching that would enable their mathematical skills to develop optimally, others seem to experience difficulties despite appropriate instruction.

In Victoria, the Department of Education and Training (DET) use the Diagnostic and Statistical Manual of Mental Disorders 5th Edition (DSM-5) to define dyscalculia thus:

> Dyscalculia is an alternative term used to refer to a pattern of difficulties characterized by problems processing numerical information, learning arithmetic facts, and performing accurate or fluent calculations. If dyscalculia is used to specify this particular pattern of mathematic difficulties, it is important also to specify any additional difficulties that are present, such as difficulties with math reasoning or word reasoning accuracy.

Dyscalculia can be contrasted with more general arithmetical difficulties as the difficulties are more likely to be persistent. According to Dowker (2004), arithmetic is multi-componential, so children can and do have difficulties with many aspects of arithmetic. Dowker also suggests that these general arithmetical difficulties may be caused by the child's environment, by maths anxiety or by inadequate instruction.

Dowker (2005) suggests that about 20% of students are likely to experience difficulty with mathematics, and Butterworth and Kovas (2013) give the estimated prevalence of dyscalculia as about 3.5-6.5% of the population. Extrapolating to Australian data, this means that there is likely to be about 117,000 students experiencing difficulties with mathematics in Victorian schools alone, with about 29,000 of these experiencing the more severe disability, dyscalculia.

One confounding factor in the diagnosis and amelioration of dyscalculia is the high co-occurrence between various specific learning disabilities (Gathercole, Woolgar, Kievit & Astlr, 2016). About 50% of students with dyslexia are also likely to have dyscalculia (Wilson & Waldie, 2010). If a child is diagnosed as having, for example, dyslexia or attention deficit hyperactivity disorder (ADHD), the difficulties the child experiences in mathematics may be assumed to be due to the dyslexia or ADHD (Butterworth & Kovas, 2013). Although intervention strategies may be implemented to remediate the students' difficulties with literacy or behaviour, it is frequently the case that interventions to remediate the dyscalculia are overlooked.

In turn, this lack of intervention can lead to poor self esteem (Williams, 2012), maths anxiety, and possible behavioural issues (Ashcraft, Krause, & Hopko, 2007; Watson & Boman, 2005). The link between learning difficulties and later delinquency has been established (Morrison & Cosden, cited in Watson & Boman, 2005), with 76% of juvenile delinquents having literacy and numeracy levels at the middle to upper primary school level.

Recommended instructional models for mathematics instruction incorporate evidence-based strategies such as Bruner's Concrete - Pictorial - Abstract (CPA) approach (Butterworth & Yeo, 2004) or the Concrete - Language - Pictorial -Symbol (CELPS) approach (Liebeck; cited in Westwood, 2000). These approaches may be beneficial for all students, including those with difficulties in mathematics.

The relative lack of awareness amongst teachers about dyscalculia and low numeracy may have a serious impact on students at a personal level and result in an economic cost to the community. This situation could be ameliorated if effective intervention strategies to help such students were used by all teachers.

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Book Review: The Dyscalculia Toolkit: Supporting Learning Difficulties in Maths (3rd Edition)

Review by **Wendy Moore**

Author: Ronit Bird, publisher: Sage

The Dyscalculia Toolkit is a practical resource (book and companion website) that can help teachers and tutors of students from 6 to 14 years to understand and ameliorate the obstacles that may impede a student's progress in mathematics. Students with dyscalculia are likely to have persistent problems with counting, quantity estimation, recalling basic facts, telling the time, learning times tables, and learning and applying formulae and rules (Soares & Patel, 2015). It is critical that schools and systems appreciate that all students, including those with mathematical learning difficulties, benefit from high expectations, a challenging curriculum, and effective instruction (Wadlington & Wadlington, 2008). In this spirit, The Dyscalculia Toolkit has a very clear purpose, which I will describe through the lens of the response to intervention model of support for students with learning difficulties.

The Why and the How of Dyscalculia Intervention

The response to intervention (RTI) model is powerful because it avoids questions about the causes of learning difficulties such as dyscalculia, and focuses instead on optimising progress through early and appropriately targeted instruction. The model assumes that students will be provided with up to three tiers of support, each of increasing intensity and specificity (Lembke, Hampton & Beyers, 2012). Dyscalculic students will likely require all three tiers to overcome their problems with understanding underlying concepts in arithmetic. The Dyscalculia Toolkit is appropriate for two of these three tiers.

The RTI model assumes that an effective and appropriately differentiated instructional program that explicitly teaches core skills and knowledge is available to every student in his or her mainstream classroom (Lembke et al., 2012). These are Tier 1 programs, and they are the bedrock of effective instruction for all students, including those with learning difficulties. Some systems and schools choose to make use of commercial mathematics programs for this purpose; others assume teachers will undertake this planning based on state or sector-wide curriculum documents. Either way, according to Wadlington and Wadlington (2008), four important principles should be applied: teachers should explain the need for practical life skills that rely on mathematical understandings so that learning has a clear purpose; teachers should acknowledge that some students experience maths anxiety, and ensure that the classroom is a safe and supportive learning environment; concrete materials should be employed as required before mathematical abstractions are presented; and effective lesson structures should be routinely used, including clear lesson objectives, step by step modelling of skills and concepts, and the generous application of guided practice and review.

The Dyscalculia Toolkit does not provide a Tier 1 learning program; the focus of this resource is much narrower than a comprehensive mathematics curriculum. Its focus is on the areas in which students with dyscalculia need the most intensive support, namely the basic arithmetical processes of number and calculation. However, many classroom teachers feel ill equipped to support students with mathematical learning difficulties and would benefit from familiarity with the main challenges that dyscalculic students experience during mathematics instruction. The insights provided by the toolkit can assist teachers in their planning and interpretation of the whole class mathematics program, including for differentiation and re-teaching within the mainstream class.

Tier 2 interventions, which complement and supplement effective classroom programs, provide additional, regular tutoring for small groups of students who have not shown expected progress (Soares & Patel, 2015). Effectively applied, Tier 2 interventions allow most students with learning difficulties to make real progress and maintain their connection to the mainstream class program (Monei & Pedro, 2017). Ideally, Tier 2 programs should run four to five times per week, for about 20 to 40 minutes per session (Lembke et al, 2012). Tier 3 interventions provide individual, targeted and sustained support for students with severe and ongoing learning difficulties who have made limited progress despite effective Tier 1 and Tier 2 programs. Individual interventions may indeed provide the most effective contexts of all for students with dyscalculia (Ise & Schulte-Köme, cited in Butterworth et al., 2011). The Dyscalculia Toolkit provides useful suggestions and resources to allow tutors or classroom teachers to develop effective Tier 2 or Tier 3 interventions for students with mathematical learning difficulties.

There are two main approaches to providing Tier 2 and Tier 3 support for students with dyscalculia (Lembke et al, 2012). One is a structured program approach, often using an evidence-based commercial package. *The Maths Mastery* series, developed in Australia by Rhonda Farkota and described in this issue, is an example of a set of scripted programs based on the principles of explicit instruction. Lembke and colleagues note that such programs, implemented with fidelity, can be highly effective in improving outcomes for low performing groups.

An alternative is a more individualised, problem-based approach which is diagnostic in design and implementation. Such an approach can target particular areas of need and focus on ensuring the development of key conceptual understandings. The approach advocated in The DyscalculiaToolkit is clearly of this second type, with a strong emphasis on diagnosis and selection of appropriate activities which are aligned to the student's area of need. The activities that are presented in the toolkit are adult-led activities and games; almost all make use of concrete materials, at least in the early stages, as a means of developing and strengthening the student's understanding of arithmetical concepts.

Does evidence support this approach?

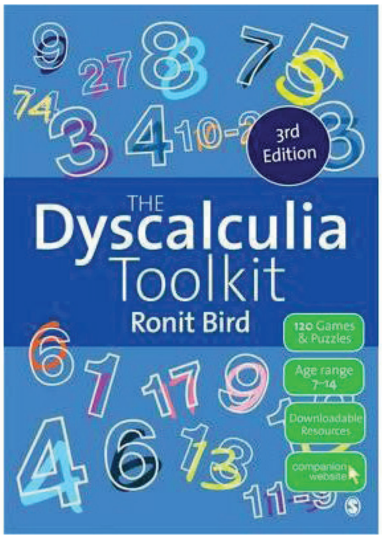
The Dyscalculia Toolkit is a collection of practical resources to support the teaching of number sense, basic fact recall and fundamental calculation strategies, designed for non-specialist teachers and tutors. The bulk of the toolkit consists of activity and game suggestions, tips and proformas for making resources, and suggestions for how they might best be sequenced and used. Bird's approach is to clarify the inadequate understandings that many students hold in four key areas, and to suggest ways to ensure that students develop both recall of facts and strategies and appropriate reasoning skills to allow them to calculate efficiently where they cannot recall. This approach is supported by existing research studies involving students with mathematical difficulties.

Rubinsten (2007) and Butterworth et al. (2011) describe a number of studies that support the efficacy of interventions which ensure a thorough emphasis on very basic number sense in a range of subtly different contexts, including games. Monei and Pedro's (2017) meta-analysis of interventions for students with dyscalculia also supports the focus on basic number sense, strategy development and frequent practice. Rubinsten (2007) offers support from neuroanatomical studies for rote learning of key number facts rather than calculation practice to circumvent persistent limitations in the ability of dyscalculic students to learn efficient computational skills. In The Dyscalculia Toolkit, Bird emphasises both: rote learning of key facts, and thorough mastery of simple mathematical reasoning underpinning calculation.

There is a focus throughout the toolkit on the use of manipulatives and games. Butterworth et al. (2011) note that the benefits of using games and concrete materials to support interventions for mathematics are threefold: providing immediate feedback on performed actions, making activities meaningful, and improving motivation. In The Dyscalculia Toolkit, Bird is at pains to ensure that the activities she has developed are child-tested and enjoyable, although, as she notes, the games are always designed with serious conceptual learning and practice in mind. The resource is divided into four main sections, and these will be reviewed in turn.

Section 1: Early number and counting

Following a brief introduction about dyscalculia, Bird begins the first main section of her book by providing an overview of the very early difficulties experienced by dyscalculic children, many of which have been resolved by their normally developing peers prior to beginning school. These include



difficulties with understanding how counting works, understanding the relative size of small numbers up to ten, recognizing the size of very small collections at a glance, and remembering simple small number bonds such as 2 + 3. She then moves quickly onto a comprehensive treatment of what to do about this.

Suggested activities include variations on games and activities that provide visual representations of number patterns using dot cards, counting pebbles, dice and dominoes. Another emphasis is the use of Cuisenaire rods to establish a sense of the relative size of small numbers and to teach number bonds and the commutative property of addition (3+5 = 5+3) without the need for counting. These suggestions are supported by photocopiable materials (both in the book itself and on the companion website, to which purchasers of the book receive access). Bird's explanations carefully interweave the how with the why, so the reader is left in no doubt about the importance of the strategies and the understandings being taught. Indeed, the author's claim that the resource is as suitable for parents or untrained tutors as it is for classroom teachers rings true because of the attention paid to the explanation of early arithmetical concepts. The instructions are helpfully specific, as this example using Cuisenaire rods demonstrates:

> Identify objects for the pupils to measure, e.g. a book or a chair. Pupils must first guess, and then measure, how many whole orange rods they can fit along the length or the height of each object. As only the 10-unit rods are being used, you can choose objects that are up to 10 rods in length or height, i.e. up to a metre.

Far from being overly prescriptive, the detailed examples are helpful in allowing the teacher or tutor to understand precisely the strategy that has been proposed. Of course, an intuitive and knowledgeable teacher will be able to adapt and expand the strategy; a novice will be confident to begin.

Section 2: Basic calculation with numbers above 10

The focus in this section is again on the thorough learning of a few key strategies, including bridging 10 and using complementary addition as a tool for subtraction. As in the previous section, concrete materials are only faded out once students have developed the requisite mental models. Indeed, the reliance on Cuisenaire rods is so strong throughout that purchasing the toolkit without access to the rods would be rather pointless. The second major conceptual tool developed in this section is the use of empty or partially completed number lines. The ability to 'see' where numbers fit on a mental number line is not only trainable, but has been shown to transfer to improved number representation and calculation (Kucian et al., 2011). As well as activities to develop concepts, this section includes instructions and proformas for a number of games for two or more players.

The range of games that this author has described confirms her real, on the ground, classroom experience. The focus of the games is clear: repeated practice to consolidate facts, with skills and strategies emphasised. The variety of games presented is in itself a demonstration to teachers that they can create effective games themselves, putting paid to the notion that commercially published materials are required or preferable.

Section 3: Place Value

Beginning with place value mat activities and hundred chart games, Bird carefully leads teachers and their students through activities to ensure that common misconceptions and inefficiencies are debunked using smaller numbers before larger numbers are introduced. She points out that place value activities should not be delayed until after the activities in the previous sections on number and calculation are addressed, but instead introduced concurrently.

The section on place value begins with the astute observation that many children have difficulty with larger numbers because they fail to understand the three-column groupings of the decimal system: three columns of *ones* (ones, tens and hundreds), three columns of *thousands* (one thousand, ten thousand, one hundred thousand), three columns of *millions* (one million, ten million, one hundred million), and so on. As a result they omit or add 'columns' in the form of zeroes when they read and write numbers, assuming that each column has its own unique name.

Like the other sections, this part has links to the companion website which provides proformas for reproduction, as well as useful short videos developed by the author which demonstrate teaching points and games. Access to the companion website requires a registration key that comes with the book. As I chose the google play (electronic) version of the toolkit, I needed to email the publishers for an access code. This process was somewhat slow, but access was simple once the code arrived some days later.

Section 4: Times tables for multiplication and division

The last section of the toolkit deals with strategies for quickly accessing multiplication and division facts. The focus is on ensuring that students understand principles and can use strategies to find answers that they might never recall by rote. This means that students are encouraged to memorise key facts, and reason from this limited set of known facts to obtain related facts. Again, learning activities, proformas and games are all provided. The author makes clear that difficulty with remembering tables should not be allowed to interfere with a dyscalculic student's ability to use reasoning to address more complex mathematical problems.

Summary

The Dyscalculia Toolkit is one of a number of books and resources written by this experienced numeracy teacher and consultant. Key areas of difficulty are briefly outlined, and carefully developed activities are provided for teachers and tutors planning Tier 2 and Tier 3 interventions for their students. The focus of the toolkit is on providing instructional strategies and resources, not on screening, progress monitoring or assessment. While a rudimentary tracking sheet is provided, decisions about the selection of starting points, the pace of introduction of new concepts, and the balance between the areas of focus are left firmly in the hands of the resource user. For teachers and tutors who know that students are having unexpected difficulties, it provides an excellent starting point for focused, diagnostic intervention. More information about this resource can be found at http://www.ronitbird.com.

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